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Biostatistics- 0504 252
Tutorial 4: Probability

Probability and Discrete Probability Distribution

Probability

Chances Have you ever asked ”what are the chances?”.

Probability The probability of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.

Example In the very long run, the proportion of tosses that give a head is 0.5. This is the intuitive idea of probability. Probability 0.5 means “occurs half the time in a very large number of trials.”

Useful Set Notation and Definitions

SET A set is a collection of objects (or elements).

INTERSECTION The joint occurrence of two sets (the intersection) contains the common elements. Notation: For the intersection of two sets, A and B, we define $A \cap B$ (sometimes, A and B).

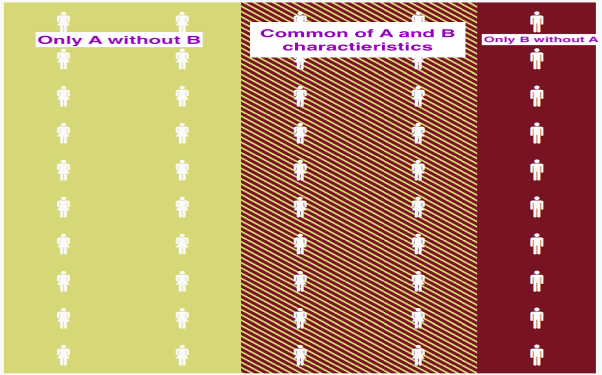
MUTUALLY EXCLUSIVE Two sets are mutually exclusive if the sets have no common elements.

UNION The union of two sets, A and B, is another set which consists of allelements belonging to set A or to set B (or some may belong to both sets). Notation: $A \cup B$.

Intersection

Example: Find the probabilities?

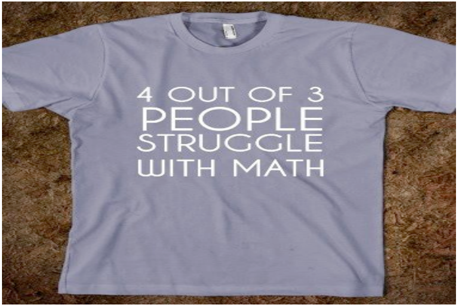
- 40 were women (characteristic A)
- 30 individuals aged 30 or older (characteristic B)
- 20 were women aged 30 years or older ($A \cap B$).
- 50 unique individuals who had one or both characteristics ($A \cup B$)



Probability Axioms

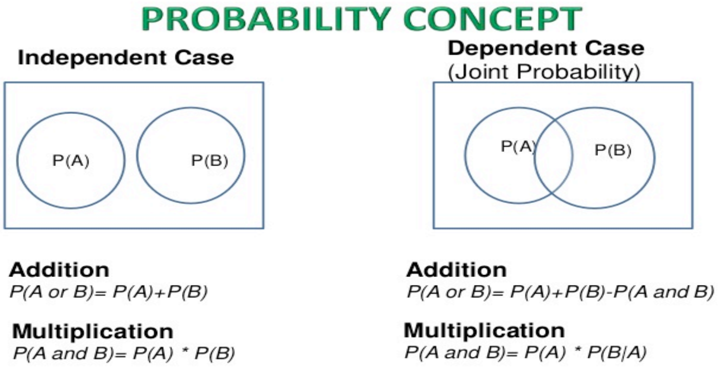
PROBABILITY OF AN EVENT A

PROPERTY 1 $0 \leq P(A) \leq 1$.
PROPERTY 2 Sum of probabilities of all events is 1.
PROPERTY 3 $P(A)^c = 1 - P(A)$.
PROPERTY 4 For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.



EXAMPLE If 7% of the population smokes cigars, 28% of the population smokes cigarettes, and 5% of the population smokes both, what percentage of the population smokes neither cigars nor cigarettes?

Probability of Events A and B



CONDITIONAL PROBABILITY

DEFN The conditional probability of an event B, given a second event A, is the probability of B happening, knowing that the event A has happened.

NOTATION Conditional probability is denoted by $P(B \mid A)$.

EXAMPLE Conditional probabilities are used in life tables.

- B = a randomly chosen person from a population will live at least 65 years
- A = a randomly chosen person will live at least 60 years
- Then $P(B \mid A)$ is the probability a person will live at least 65 years, given that they have been selected from the population of people 60 years of age or older.
- $P(B \mid A) = \frac{P(AB)}{P(A)}$.

Calculating Probability

EXAMPLE 1: RELATIONSHIP BETWEEN GENDER AND AGE

Patients with Disease X			
	Age		
Gender	Young	Older	Total
Male	30	20	50
Female	40	110	150
Total	70	130	200

- P(Male) = 50/200 = 0.25
- P(Female) = 150/200 = 0.75
- P(Young) = 70/200 = 0.35
- P(Older) = 130/200 = 0.65
- Joint probability, P(Female and Older)= 110/200= 0.55
- P(Female or Older)=? (Ans: 0.85)

Conditional Probability

Example 2

All the students in a certain high school were surveyed, then classified according to gender and whether they had either of their ears pierced:

	Pierced	Not Pierced	Total
Male	36	144	180
Female	288	32	320
Total	324	176	500

(Note that this is a two-way table of counts that was first introduced when we talked about the relationship between two categorical variables.

It is not surprising that we are using it again in this example, since we indeed have two categorical variables here:

- **Gender:**M or F (in our notation, "not M")
- **Pierced:**Yes or No

Suppose a student is selected at random from the school.

- Let **M**and **not M** denote the events of being male and female, respectively,
- and **E**and **not E** denote the events of having ears pierced or not, respectively.

What is the probability that the student has either of their ears pierced?

Since a student is chosen at random from the group of 500 students, out of which 324 are pierced,

- **P(E) = 324/500 = 0.648**

Conditional Probability

Example 2- Continue

- What is the probability that the student is male? (P(M) = 180/500 = 0.36).
- What is the probability that the student is male and has ear(s) pierced? (P(M and E) = 36/500 = 0.072).
- Given that the student that was chosen is male, what is the probability that he has one or both ears pierced? ($P(E|M) = 36/180 = 0.20$).

Conditional Probability

Example 3

On the "Information for the Patient" label of a certain antidepressant, it is claimed that based on some clinical trials,

- there is a 14% chance of experiencing sleeping problems known as insomnia (denote this event by **I**),
- there is a 26% chance of experiencing headache (denote this event by **H**),
- and there is a 5% chance of experiencing both side effects (**I and H**).

(a) Suppose that the patient experiences insomnia; what is the probability that the patient will also experience headache?

Since we know (or it is **given**) that the patient experienced **insomnia**, we are looking for **P(H | I)**.

According to the definition of conditional probability:

- **P(H | I) = P(H and I) / P(I) = 0.05/0.14 = 0.357.**

(b) Suppose the drug induces headache in a patient; what is the probability that it also induces insomnia?

Here, we are given that the patient experienced headache, so we are looking for P(I | H).

Using the definition

- **P(I | H) = P(I and H) / P(H) = 0.05/0.26 = 0.1923.**

Probability

Some Notes

Randomness We call a phenomenon random if individual outcomes are uncertain but there is nonetheless a regular distribution of outcomes in a large number of repetitions.

Independent trials The outcome of one trial must not influence the outcome of any other.

Important Note The idea of probability is empirical. Simulations start with given probabili- ties and imitate random behavior, but we can estimate a real-world probabil- ity only by actually observing many trials.

Exercise You go to the doctor and she prescribes a medicine for an eye infection that you have. Suppose that the probability of a serious side effect from the medicine is 0.00001. Explain in simple terms what this number means.

Probability

Some Notes

Random Variable A random variable is a variable whose value is a numerical outcome of a random phenomenon. The probability distribution of a random variable X tells us what the possible values of X are and how probabilities are assigned to those values.

Discrete Random Variable A discrete random variable has finitely many possible values. The probability distribution assigns each of these values a probability between 0 and 1 such that the sum of all the probabilities is exactly 1. The probability of any event is the sum of the probabilities of all the values that make up the event.

Continuous Random Variable A continuous random variable takes all values in some interval of numbers. A density curve describes the probability distribution of a continuous random variable. The probability of any event is the area under the curve and above the values that make up the event.

Discrete probability distributions

EXAMPLE Hyponatremia (low sodium levels) occurs in a certain proportion of marathon runners. Now the question is, how many cases of hyponatremia are expected during the running of a particular marathon?

What do you need to know to answer this question?

- Suppose there are 200 runners participating in a marathon (n = 200).
- Suppose historically the proportion of runners who develop hyponatremia is assumed to be 0.12 (p = 0.12).
- Let X denote the number of these runners who develop hyponatremia. Then X follows **binomial random variable**.

Probability Distribution

Mean and Variance

Mean or Expectation, μ_X $E(X) = \sum x_j f_X(x_j)$. or $E(X) = \int_{-\infty}^{\infty} x f_X(x) dx$.

Variance, σ_X^2 $V(X) = E[(x - E(X))^2]$.

Standard Deviation SD is the square root of the estimated variance.

Rules of Means :

- If X is a random variable and a and b are fixed numbers, then

$$\mu_{a+bX} = a + b\mu_X$$

- If X and Y are random variables, then

$$\mu_{X+Y} = \mu_X + \mu_Y$$

Rules of Variances Find out.